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Fuzzy optimization in hydrodynamic analysis of groundwater control systems: Case study of the pumping station "Bezdan 1", Serbia

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Abstract. A groundwater control system was designed to lower the water table and allow the pumping station "Bezdan 1" to be built. Based on a hydrodynamic analysis that suggested three alternative solutions, multicriteria optimization was applied to select the best alternative. The fuzzy analytic hierarchy process method was used, based on triangular fuzzy numbers. An assessment of the various factors that influenced the selection of the best alternative, as well as fuzzy optimization calculations, yielded the "weights" of the alternatives and the best alternative was selected for groundwater control at the site of the pumping station "Bezdan 1".

Key words: groundwater lowering, groundwater management scenario, fuzzy analytic hierarchy process, expert knowledge, triangular fuzzy numbers, linguistic variables.

Апстракт. За обарање нивоа подземних вода како би се изградила црпна станица „Бездан 1" пројектован је систем одбране од подземних вода. На основу хидродинамичке анализе којом су дефинисане три алтернативе решења система, приступило се методом вишекритеријумске оптимизације како би се одабрала оптимална алтернатива. За те потребе коришћена је метода фази аналитичко хијерархијског процеса, базирана на троугластим фази бројевима. Анализом различитих фактора који утичу на избор алтернативе и фази оптимизационим прорачунима, добијене су „тежине" алтернатива и донета је одлука о оптималној алтернативи решења система одбране од подземних вода на подручју црпне станице „Бездан 1".

Кључне речи: обарање нивоа подземних вода, варијанте система одбране од подземних вода, фази аналитичко хијерархијски процес, знање експерта, троугаони fuzzy број, лингвистичка варијабла.

Introduction

The best way engineers or scientists can express their opinions is in fact everyday verbal communication. It is a significant source of uncertainty, because of the transfer of both information and knowledge coupled with various uncertainty and imprecision (Kosko 1993). This is the reason why fuzzy logic systems are distinguished, given that their essence is to handle knowledge that can be highly imprecise and expressed verbally. In fuzzy logic systems this "knowledge" is represented by production (expert) rules, which are a suitable verbal means of expressing the knowledge of each individual. Consequently, fuzzy logic is the codification of common sense (Graham 1991; Lai & Hwang 1996). Expert knowledge is used instead of differential equations to describe a system. The knowledge is conveyed in a natural way, by linguistic variables, such that fuzzy logic is "computing with words" (Zadeh 1965; Zadeh 1975).

As indicated above, the basic unit that represents knowledge in fuzzy logic is a linguistic variable, with its linguistic values that make up fuzzy sets. Combinations of variables and their values produce linguistic statements (expressions), which constitute a bridge between numerical representation of information on a computer and human thinking. Zadeh (1975) introduced the "linguistic (fuzzy) variable", which is the value of an uncertainty described by a linguistic statement. The linguistic or fuzzy variable is

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defined as the variable whose permissible values are words of natural language, and not numbers. Apart from their symbolic linguistic form, linguistic variables also have a quantified analytical form-membership function, such that they are of a dual nature. This dual identity makes linguistic variables suitable for qualitative-symbolic and quantitative-numerical calculations. A correlation is thereby established between the natural language used by man and the numerical data used by a computer.

In recent years there has been a rapid increase in the number and types of applications of systems based on the fuzzy theory. While it was originally used to analyze data that allow for partial set membership, to avoid the approach where there is either full or no such membership, today fuzzy logic is a management method which, despite its early stage, is increasingly used in all fields of science, including hydrogeology - groundwater management, water quality management, dewatering and groundwater control, etc. (AZARNIVAND *et al.* 2004; SINGH *et al.* 2007; UDDAMERI *et al.* 2014; KARNIB 2014).

One branch of fuzzy logic applied in hydrogeology is fuzzy optimization. The present paper describes an application of multicriteria fuzzy optimization - the so-called fuzzy analytic hierarchy process (FAHP), to select the optimal groundwater management scenario (groundwater control system) for the pumping station "Bezdan 1" ("Bezdan 1" PS), out of three alternatives derived from the hydrodynamic analysis reported by POLOMČIĆ & BAJIĆ (2014).

There are many FAHP approaches suggested by different authors in the literature. LAARHOVEN & PEDRCYZ (1983) initiated the first studies that applied fuzzy logic and the analytic hierarchy process. They used triangular fuzzy numbers to express the evaluation by the decision maker (expert) of alternatives against each of the given criteria, while BUCKLEY (1985) used trapezoidal fuzzy numbers for the same purpose. CHANG (1996) introduced a new approach to FAHP, using triangular fuzzy numbers, FAHP scale and extent analysis to compare pairs of criteria in a matrix. Going into more detail, CHAN & KUMAR (2007) added risk factors to the extent analysis in FAHP, which included uncertain information used in decision making. DENG (1999) proposed a fuzzy approach to solving the problem of qualitative multicriteria analysis - using FAHP for multicriteria decision making. ZHU *et al.* (1999) presented the fundamental theories of fuzzy triangular numbers, contributed an improved approach to their comparison, and demonstrated a practical application to an oil field. CHOU & LIANG (2001) proposed a fuzzy multicriteria decision-making concept that integrated the fuzzy theory with the analytic hierarchic process. Additionally, CHANG *et al.* (2003) introduced statistical methods to FAHP, for selecting the criterion that affected the end result (i.e. selection of alternative). Among the FAHP approaches mentioned above, the one devised by CHANG (1996) was used to arrive at the best alternative for the groundwater control system of the "Bezdan 1" PS.

Study Area

To prevent an environmental disaster in the Bačka District (Serbia), as well as avoid suspensions of navigation and different types of water supply, it is necessary to expand the Bačka section of the Danube–Tisa–Danube Water Scheme. To resolve one of the functional issues of the multipurpose Danube–Tisa–Danube system, new pumping stations (total capacity 35 m^3/s) will have to be built at the locations of previous pumping stations, which have been out of commission for some time. Apart from dealing with the issues mentioned above, this project will regulate the Bezdan–Vrbas Canal, the Baračka Canal and a number of smaller canals. The present research addresses the "Bezdan 1" PS, located in the Town of Bezdan in northwestern Bačka. Figure 1 shows the geographical position and microlocation of the study area.

Predictive Hydrodynamic Analysis

The site of the future "Bezdan 1" PS features high groundwater levels, which directly affect the feasibility of construction. Three-dimensional (3D) hydrodynamic modeling, in this case to define a groundwater control system for the "Bezdan 1" PS, is a common approach in modern hydrogeology. The hydrogeological setting was schematized, the input parameters defined and a model based on finite differences constructed to develop a predictive hydrodynamic analysis after matching natural and modeled hydrogeological parameters through calibration. As a result, POLOMČIĆ & BAJIĆ (2014) proposed a groundwater control system for the "Bezdan 1" PS, whose function is to lower the water table to below the design elevation, so that the pumping station can be built. Three alternatives were considered, to ensure protection against groundwater intrusion and provide for unhindered construction of the "Bezdan 1" PS. Figure 2 shows the lowering of the water table of the analyzed alternatives. For each alternative, the characteristics of the groundwater control system: the number of wells and their spatial distribution, and the time needed to achieve maximal lowering of the water table below the excavation for the "Bezdan 1" PS, were defined as follows:

- Alternative 1 (A_1): Long-term average stages of the canals within the study area (Baračka Canal 82.80 m above sea level and Bezdan–Vrbas Canal 84.80 m.a.s.l.) were used in the predictive hydrodynamic analysis. The groundwater control sys-

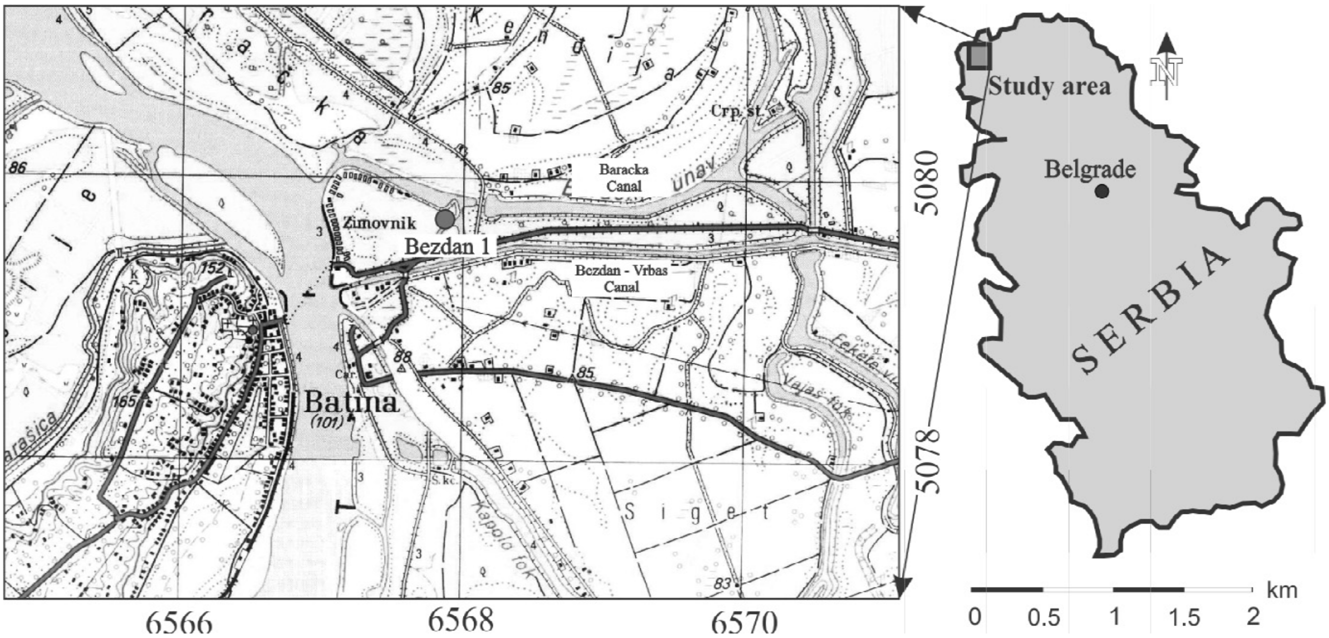

Fig. 1. Study area and "Bezdan 1" pumping station.

tem is comprised of 12 wells, whose capacity is 40 l/s each. The time needed to lower the water table to below the design elevation and allow for the construction work to proceed is 5 days.

- Alternative 2 (A_2): Long-term average maximum stages of the canals within the study area (Baračka Canal 86.61 m.a.s.l. and Bezdan–Vrbas Canal 85.16 m.a.s.l.) were used in the predictive hydrodynamic analysis. The groundwater control system is comprised of 12 wells, whose capacity is 40 l/s each. The water table "stabilizes" after 7 days of operation of the system.
- Alternative 3 (A_3): In this case, lowering of the water table was simulated using maximum elevations of the designed cofferdams for the following canal stages: Baračka Canal 87 m.a.s.l. and Bezdan–Vrbas Canal 86.5 m.a.s.l. The groundwater control system is comprised of 17 wells, whose capacity is 40 l/s each. The time needed to lower the water table to below the design elevation is 8.5 days.

Based on the predictive analysis and the three identified alternatives for the groundwater control system, as described in Polomčić & Bajić (2014), the selection of the best alternative applying the FAHP approach is discussed below.

Fuzzy Optimization Method

Apart from the several approaches to fuzzy optimization and the use of the FAHP approach described

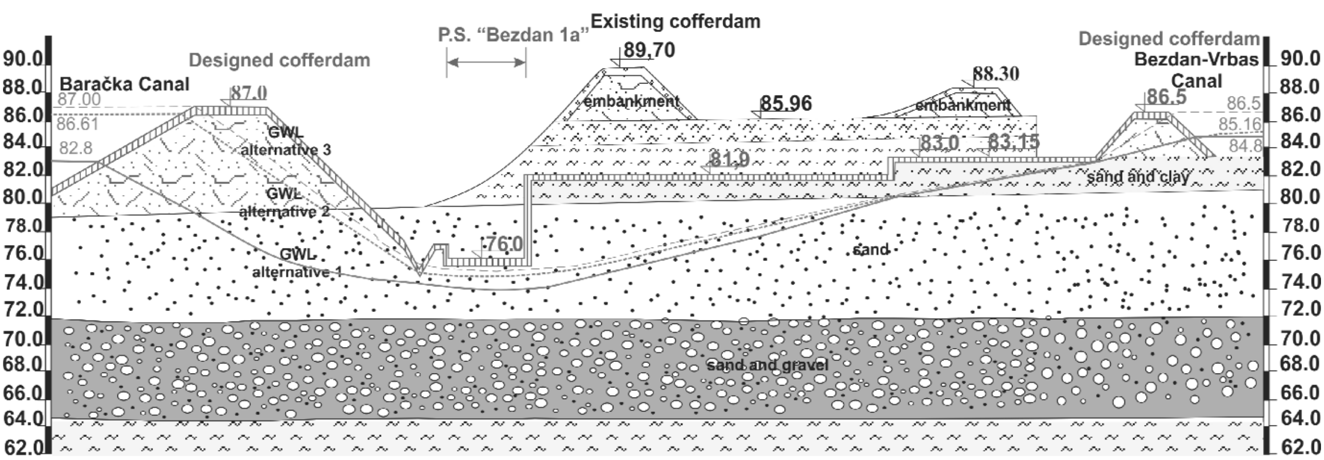

Fig. 2. Lowering of the water table, alternatives 1, 2 and 3, as a result of control system operation.

above, the solution produced by hydrodynamic calculations was optimized by means of fuzzy extent analysis, an FAHP approach proposed by (CHANG 1996). This method is based on triangular fuzzy numbers and Saaty's pairwise comparison scale (SAATY 1980). The main features of this method are presented below in several steps.

The basic concept of the FAHP approach is the triangular fuzzy number. If this number is denoted by $M(l,s,d)$, as illustrated in Fig. 3, it is defined by its membership function as follows:

$$\mu_M(x) = \begin{cases} \dfrac{x}{s-l} - \dfrac{l}{s-l}, & x \in [l, s] \\ \dfrac{x}{s-d} - \dfrac{l}{s-d}, & x \in [s, d] \quad \text{where is } l \le s \le d \\ 0, & x \notin [l, d] \end{cases}$$

The correlation between the numerical values of triangular fuzzy numbers and linguistic variables is represented by fuzzified Saaty's scale (CHANG 1996; DENG 1999; TOLGA 2005). One such correlation is shown in Table 1.

Table 1. Fuzzified Saaty's scale for pairwise comparisons (DENG 1999)

Linguistic variable (judgment definition)	Saaty's crisp value	FAHP scale Triangular fuzzy numbers ($0.5 \le \alpha \le 0.5$)
Equal importance	1	(1, 1, 1+α)
Weak dominance	3	(3–α, 3, 3+α)
Strong dominance	5	(5–α, 5, 5+α)
Demonstrated dominance	7	(7–α, 7, 7+α)
Absolute dominance	9	(9–α, 9, 9)
Intermediate values	2, 4, 6, 8	(x–1, x, x+1) x=2, 4, 6 ,8

Step 1. Taking the pre-defined factors that affect the selection of one among several alternatives, a matrix of criterion X is constructed with triangular fuzzy numbers assigned by the decision maker (expert), using the FAHP scale.

Step 2. Taking the generated matrix, an extent analysis of all the elements of the matrix is conducted, resulting in m values of step analyses for each element of the set X:

$$M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m, \; i = 1, 2, \ldots, n,$$

where all $M_{g_i}^{\,j}$, $j = 1, 2, \ldots, m$ are triangular fuzzy numbers.

Then, taking into account the membership function of the triangular fuzzy numbers, the value of the fuzzy synthetic extent is computed using the expression:

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} =$$

$$= \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} s_j, \sum_{j=1}^{m} d_j \right) \otimes \left(\frac{1}{\sum_{i=1}^{n} d_j}, \frac{1}{\sum_{i=1}^{n} s_j}, \frac{1}{\sum_{i=1}^{n} l_j} \right)$$

Step 3. In this step the degree of possibility of two triangular fuzzy numbers (Fig. 3) $M_1 = (l_1, s_1, d_1)$ and $M_2 = (l_2, s_2, d_2)$, is determined applying the fuzzy number comparison principle:

$$V(M_1 \ge M_2) = \sup_{x \ge y} \left[\min\left(\mu_{M_1}(x), \mu_{M_2}(y) \right) \right]$$

If there are pairs (x, y) such that $x \ge y$ and $\mu_{M_1}(x) = \mu_{M_2}(y) = 1$, then $V(M_1 \ge M_2) = 1$. Since M_1 and M_2 are convex triangular fuzzy numbers, it follows that:

$V(M_1 \ge M_2) = 1$ *if* $s_1 \ge s_2$

$$V(M_2 \ge M_1) = hgt(M_1 \cap M_2) = \mu_{M_1}(c) \begin{cases} 1, & \text{if } s_2 \ge s_1 \\ 0 & \text{if } l_1 \ge d_2 \\ \dfrac{l_1 \quad d_2}{(s_2 - d_2) - (s_1 - l_1)}, & other \end{cases}$$

where c is the ordinate of the highest intersection at point C between the membership functions μ_{M_1} and μ_{M_2}

To compare the triangular fuzzy numbers M_1 and M_2, both values, $V(M_1 \ge M_2)$ and $V(M_2 \ge M_1)$ are needed.

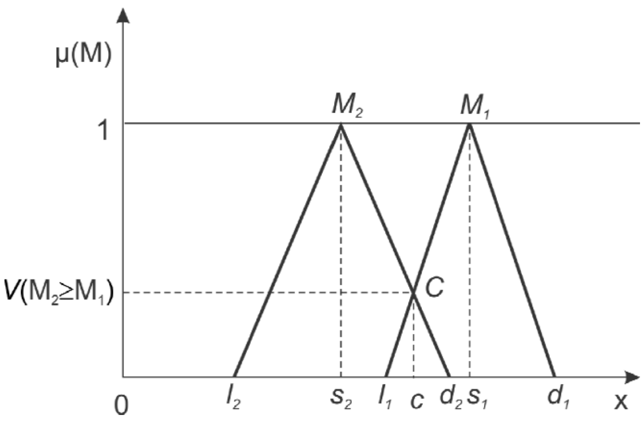

Fig. 3. Triangular fuzzy number.

The degree of possibility of a convex fuzzy number to be greater than k, the convex fuzzy numbers M_i can be defined by i=1,2,…,k:

$$V(M \geq M_1, M_2, ...M_k) = V[(M \geq M_1) \wedge ... \wedge (M \geq M_k)] =$$
$$= \min V(M \geq M_i)$$

Then, summing everything up, it follows that: $c'(A_i)=\min V(S_i \geq S_k)$, $k=1,2,\ldots, n$; $k \neq i$

Step 4. Continuing from the previous step, the weight priority vectors are defined as follows:

$$W' = \left(c'(A_1), c'(A_2), ..., c'(A_n)\right)^T \text{ where } A_i \ (i=1,2,\ldots,n)$$

Step 5. The ultimate values of the weights are obtained applying one of the normalization methods:

- The additive normalization method (Saaty 1980), or
- The weighted least squares method (Chu *et al.* 1979), the logarithmic least squares method (Crawford & Williams 1985), the fuzzy preference programming method (Mikhailov 2000), the direct least squares method (Chu *et al.* 1979), which often yields multiple solutions, or the logarithmic goal programming method (Bryson 1995).

This results in a normalized weight vector in the form of a classical non-fuzzy number, whose maximum value is 1:

$$W = \left(c(A_1), c(A_2), ..., c(A_n)\right)^T$$

Step 6. In this step the alternatives are compared for each criterion separately. Matrices are produced and then the weight vectors determined following Steps 1 through 5.

Step 7. Here the ultimate weights of the alternatives are determined. They are obtained by multiplying the weight vectors derived from the criterion matrix by the weight vectors from Step 6. The alternative with the greatest weight vector value is the best alternative.

Results and Discussion

The fuzzy extent analysis and the analytic hierarchic process described above were applied to develop a decision-making model and select the best alternative for the groundwater control system of the "Bezdan 1" PS. Fuzzy optimization calculations included assessments of the various factors/criteria that affected the selection of the best possible solution to the problem. In general, it is difficult to produce an alternative that will instantaneously satisfy all the applicable criteria, but an acceptable trade-off can be found. The following criteria were considered in connection with the present groundwater control system:

- *Time (K_1),* which is the time needed for the water table to be lowered to the design level. To assess this criterion, it was not assumed that the best time was the shortest time. Instead, the time was related to the analyzed conditions and the elevation of the water table for each alternative.
- *Characteristics of the groundwater control system (K_2),* which included the number of components of the groundwater control system, where the system was analyzed and its cost-effectiveness and flexibility assessed. In the present study, as the number of wells increased, so did capital expenditure and operating and maintenance costs. However, it should be noted that in some cases the construction of wells is more economical than the pumping time. Flexibility included the ability of the system to adapt to possible changes to its characteristics. Consequently, when the time comes to lower the water table, if the groundwater levels are below those used in the hydrodynamic analysis of the selected alternative, not all the wells need to be constructed or operated.
- *Safety factor (K_3)*, which represents an analysis of possible water table conditions at the time of lowering for the purposes of constructing the "Bezdan 1" PS. This analysis was used to assess the status of boundary conditions in the hydrodynamic analysis of different canal stages in the study area.

Shown below are the calculations for the selection of the best alternative in accordance with the above-described steps (the FAHP approach discussed in the previous section).

The criteria matrix was produced by evaluating the criteria according to the fuzzified scale (TOLGA 2005):

$$X = \begin{bmatrix} & K_1 & K_2 & K_3 \\ K_1 & 1,1,1 & \frac{2}{3},1,2 & \frac{1}{2},1,\frac{3}{2} \\ K_2 & \frac{1}{2},1,\frac{3}{2} & 1,1,1 & 1,\frac{3}{2},2 \\ K_3 & \frac{2}{3},1,2 & \frac{1}{2},\frac{2}{3},1 & 1,1,1 \end{bmatrix}$$

Using a specially developed application, according to Step 2, the calculated values of the fuzzy synthetic extent were as follows:

$$S_1 = (2.16, 3, 4.5) \otimes \left(\frac{1}{13}, \frac{1}{9.16}, \frac{1}{6.82}\right) = (0.17, 0.33, 0.66)$$

$$S_2 = (2.5, 3.5, 4.5) \otimes \left(\frac{1}{13}, \frac{1}{9.16}, \frac{1}{6.82}\right) = (0.19, 0.38, 0.66)$$

$$S_3 = (2.16, 2.66, 4) \otimes \left(\frac{1}{13}, \frac{1}{9.16}, \frac{1}{6.82}\right) = (0.17, 0.29, 0.59)$$

According to Step 3, the fuzzy numbers were compared and the degree of possibility of two triangular fuzzy numbers (i.e. their values of fuzzy synthetic extent) was determined as follows:

$$V\left(S_1 \geq S_2\right) = \frac{0.19-0.66}{\left(0.33-0.66\right)-\left(0.38-0.19\right)} = 0.9$$

$$V\left(S_1 \geq S_3\right) = 1$$

$$V\left(S_2 \geq S_1\right) = 1$$

$$V\left(S_2 \geq S_3\right) = 1$$

$$V\left(S_3 \geq S_1\right) = \frac{0.17-0.59}{\left(0.29-0.59\right)-\left(0.33-0.17\right)} = 0.91$$

$$V\left(S_3 \geq S_1\right) = \frac{0.19-0.59}{\left(0.29-0.59\right)-\left(0.38-0.19\right)} = 0.81$$

Then the minimum value was selected, as described in Step 3, and the weight vectors obtained as:

W' = (0.9,1,0.81)

Finally, the ultimate criterion weights were obtained through normalization:

W = (0.33, 037, 0.3)

Shown below are the criterion - based evaluation of alternatives (where three matrices were generated) and the ultimate weight vectors for each comparison.

Criterion 1 (matrix, the value of fuzzy synthetic extent, the degree of possibility, the weight priority vectors and the ultimate values of the weights):

$$X_{K_1} = \begin{bmatrix} K_1 & A_1 & A_2 & A_3 \\ A_1 & 1,1,1 & 4,5,6 & \frac{1}{5},\frac{1}{4},\frac{1}{3} \\ A_2 & \frac{1}{6},\frac{1}{5},\frac{1}{4} & 1,1,1 & 6,7,8 \\ A_3 & 3,4,5 & \frac{1}{8},\frac{1}{7},\frac{1}{6} & 1,1,1 \end{bmatrix}$$

$$S_{1K_1} = \left(5.2, 6.25, 7.33\right) \otimes \left(\frac{1}{22.74}, \frac{1}{19.59}, \frac{1}{16.48}\right) = \left(0.23, 0.32, 0.44\right)$$

$$S_{2K_1} = \left(7.16, 8.2, 9.25\right) \otimes \left(\frac{1}{22.74}, \frac{1}{19.59}, \frac{1}{16.48}\right) = \left(0.31, 0.42, 0.56\right)$$

$$S_{3K_1} = \left(4.12, 5.14, 6.16\right) \otimes \left(\frac{1}{22.74}, \frac{1}{19.59}, \frac{1}{16.48}\right) = \left(0.18, 0.26, 0.37\right)$$

$$V_{K_1}\left(S_1 \geq S_2\right) = 0.56 \quad V_{K_1}\left(S_1 \geq S_3\right) = 1$$

$$V_{K_1}\left(S_2 \geq S_1\right) = 1 \quad V_{K_1}\left(S_2 \geq S_3\right) = 1$$

$$V_{K_1}\left(S_3 \geq S_1\right) = 0.7 \quad V_{K_1}\left(S_3 \geq S_2\right) = 0.01$$

W' = (0.56,1,0.01)
W = (0.36,0.64,0.01)

Criterion 2 (matrix, the value of fuzzy synthetic extent, the degree of possibility, the weight priority vectors and the ultimate values of the weights):

$$X_{K_2} = \begin{bmatrix} K_2 & A_1 & A_2 & A_3 \\ A_1 & 1,1,1 & 5,6,7 & \frac{1}{8},\frac{1}{7},\frac{1}{6} \\ A_2 & \frac{1}{7},\frac{1}{6},\frac{1}{5} & 1,1,1 & 7,8,9 \\ A_3 & 6,7,8 & \frac{1}{9},\frac{1}{8},\frac{1}{7} & 1,1,1 \end{bmatrix}$$

$$S_{1K_2} = \left(6.12, 7.14, 8.16\right) \otimes \left(\frac{1}{27.5}, \frac{1}{24.42}, \frac{1}{21.37}\right) = \left(0.22, 0.29, 0.38\right)$$

$$S_{2K_2} = \left(8.14, 9.16, 10.2\right) \otimes \left(\frac{1}{27.5}, \frac{1}{24.42}, \frac{1}{21.37}\right) = \left(0.3, 0.38, 0.48\right)$$

$$S_{3K_2} = \left(7.11, 8.12, 9.14\right) \otimes \left(\frac{1}{27.5}, \frac{1}{24.42}, \frac{1}{21.37}\right) = \left(0.26, 0.33, 0.43\right)$$

$$V_{K_2}\left(S_1 \geq S_2\right) = 0.06 \quad V_{K_2}\left(S_1 \geq S_3\right) = 1$$

$$V_{K_2}\left(S_2 \geq S_1\right) = 1 \quad V_{K_2}\left(S_2 \geq S_3\right) = 1$$

$$V_{K_2}\left(S_3 \geq S_1\right) = 1 \quad V_{K_2}\left(S_3 \geq S_2\right) = 0.72$$

W' = (0.06,1,0.72)
W = (0/03,0.56,0.4)

Criterion 3 (matrix, the value of fuzzy synthetic extent, the degree of possibility, the weight priority vectors and the ultimate values of the weights):

$$X_{K_3} = \begin{bmatrix} K_3 & A_1 & A_2 & A_3 \\ A_1 & 1,1,1 & 6,7,8 & \frac{1}{9},\frac{1}{9},\frac{1}{8} \\ A_2 & \frac{1}{8},\frac{1}{7},\frac{1}{6} & 1,1,1 & 4,5,6 \\ A_3 & 8,9,9 & \frac{1}{6},\frac{1}{5},\frac{1}{4} & 1,1,1 \end{bmatrix}$$

$$S_{1K_3} = \left(7.11, 8.11, 9.12\right) \otimes \left(\frac{1}{26.53}, \frac{1}{24.45}, \frac{1}{21.39}\right) = \left(0.27, 0.33, 0.43\right)$$

$$S_{2K_3} = \left(5.12, 6.14, 7.16\right) \otimes \left(\frac{1}{26.53}, \frac{1}{24.45}, \frac{1}{21.39}\right) = \left(0.19, 0.25, 0.33\right)$$

$$S_{3K_3} = \left(9.16, 10.2, 10.25\right) \otimes \left(\frac{1}{26.53}, \frac{1}{24.45}, \frac{1}{21.39}\right) = \left(0.35, 0.42, 0.48\right)$$

$$V_{K_3}\left(S_1 \geq S_2\right) = 1 \quad V_{K_3}\left(S_1 \geq S_3\right) = 0.47$$

$$V_{K_3}\left(S_2 \geq S_1\right) = 0.42 \quad V_{K_3}\left(S_2 \geq S_3\right) = 0.13$$

$$V_{K_3}\left(S_3 \geq S_1\right) = 1 \quad V_{K_3}\left(S_3 \geq S_2\right) = 1$$

W' = (0.47,0.13,1)
W = (0.29,0.08,0.63)

Based on these calculations, the ultimate evaluation of the alternatives is shown in Table 2.

The criteria-based assessment showed that the best solution for the groundwater control system of the "Bezdan 1" PS was Alternative 2. Here the control system is comprised of 12 wells. It takes seven days to lower the water table to below the design elevation and establish quasi-steady groundwater flow. As pointed out in connection with the evaluation of criteria, in this alternative, if the canal stages should be

Table 2. Ultimate evaluation of alternatives.

Criterion	Criterion weight	Alternative 1	Alternative 2	Alternative 3
1	0.33	0.36	0.64	0.01
2	0.37	0.03	0.56	0.4
3	0.3	0.29	0.08	0.63
Final score:		0.22	0.44	0.34

below the long-term average maximum levels, some of the wells can be shut down, as needed.

Conclusion

Human beings are often uncertain in assigning evaluation scores by conventional methods, so decision making frequently involves uncertainties. FAHP can cope with that difficulty. The method has the ability to capture the vagueness of human thinking and effectively solve multicriteria decision-making problems. In the present study, the FAHP approach was applied to assess the factors that affect the selection of the optimal groundwater control system at the location of the future pumping station "Bezdan 1", whose construction is hindered by high groundwater levels. Applying this multicriteria approach, three alternatives, previously identified by hydrodynamic modeling, were assessed. According to the results of the FAHP analysis, each of the alternatives was evaluated and Alternative 2 was found to be the "best". It scored 0.44/1. As a result, Alternative 2 was proposed for the groundwater control system associated with the "Bezdan 1" PS, whose general and spatial characteristics are: 12 wells with individual capacity of 40 l/s and seven days to lower the water table.

In some cases, the FAHP analysis can eliminate certain criteria, assigning them weights close to zero. Such clustering may help managers make decisions based on the most important criteria, especially in cases where more precise information can be expensive to obtain. Proposed for future study are other multicriteria optimization methods and decision-making approaches, to compare several different methods and present the results in parallel. The FAHP analysis used in the present study can be recommended for other fields of science and technology, where optimization is required and the best-possible decision needed.

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Резиме

Примена fuzzy оптимизације у хидродинамичкој анализи за потребе избора система одбране од подземних вода: пример црпне станице “Бездан 1”, Р. Србија

На основу предходних прогнозних хидродинамичких прорачуна, како би се оборио ниво подземних вода, дефинисане су три варијанте (алтернативе) решења система одбране од подземних вода и његове карактеристике којим се штити подручје будуће црпне станице „Бездан 1“ како би се она могла изградити.

Коришћењем фуззу оптимизације - методе fuzzy аналитичко хијерархијског процеса, базиране на троугаоним fuzzy бројевима и анализом различитих фактора као што су време које је потребно да се ниво подземних вода спусти на проектовани ниво, затим карактеристике система одбране од подземних вода и фактор сигурности, дате су различите оцене које утичу на избор оптималне алтернативе.

Према прорачунима ове методе вишекритеријумске оптимизације, анализиране су три алтернативе решења система одбране од подземних вода, које су утврђене раније, хидродинамичким моделирањем. На основу добијених резултата фази аналитичко хијерархијских прорачуна, добијене су оцене за сваку алтернативу, где се показало да је „најбоља” алтернатива број два, са оценом 0.44/1.

Као крајње решење постављеног проблема, од три понуђене алтернативе дефинисане хидродинамичким прорачунима, одабран је оптималан систем одбране од подземних вода црпне станице „Бездан 1”, тзв. „Алтернатива 2” која укључује у систем одбране 12 бунара појединачних капацитета од 40 l/s, где је потребно 7 дана да се обори ниво подземних вода.